VEDIC MATHEMATICS COURSE

CHINMAYA INTERNATIONAL FOUNDATION

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PART I

INTRODUCTION

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Mathematics is an amazing subject. It is the queen of all sciences. Yet, it is a horrifying word for many across the world. It has been considered as a dry subject, studied as an obligation. The application of Mathematics in subjects like Physics, Chemistry, Astronomy, Accountancy, Statistics, Geography and even Economics, has left people with no choice but to learn it, whether they like it or not.

School children feel that they have to deal with Mathematics only till they finish school. When they enter college, this dream is often shattered. Things do not end there. After graduation, when they appear for competitive exams or an aptitude test for a corporate job, their plight gets worse. After having got habituated to using calculators all these years, they find it hard even to add a list of numbers without its help. Many youngsters are not able to crack these exams only because of their challenge in numerical ability. What a tragedy would it be for those who are unable to pursue their dream career and have to be satisfied with what they have got!

Let us face it. Mathematics is a subject that chases us like a shadow in our school, college and professional life. How to deal with such a problem? Many researchers have pondered and worked towards solving this 'mathematical' puzzle. And this is what they have concluded – The subject is more like a phobia that gets inflicted in one's mind at a young age. It can be a fear for a particular teacher that got converted to a fear of the subject. It can also be a general perception that 'Mathematics is hard' that got stuck in

our mind during school days. Whatever the reason be, it is just a 'fear' that can be overcome with ease. This programme aims to change that perception. Mathematics can be fun!

Mathematics in Ancient India

In the olden times in India, mathematics was not taught as a separate subject. It was imbibed in other subjects like Astrology, Astronomy, Engineering, and so on. This added a different flavor to mathematics. Students were made to observe the forming of different shapes and patterns in the nature. They were shown how different numbers had different properties as could be seen in any field – be it science, spirituality, astrology and many more.

Today, we learn mathematics as a separate subject without realizing its real application in other subjects. When we take mathematics out of any subject and treat it separately, the sensse of versatality is gone. It is like eating raw tea leaves rather than having it with water, sugar and milk. This may be one of the reasons why mathematics is often looked at as a dry subject. But if we look carefully, we can see the play of mathematics in the entire Universe. Take any subject – Music, Dance, Literature, Poetry, Physics, Chemistry, Biology, Engineering, Economics, Statistics, Accountancy or Computers; every subject has mathematical applications. Mathematics is the unseen thread that links various subjects together to form a beautiful necklace of the Total Knowledge.

Indian Mathematics can be broadly classified into four categories:

1. Mathematics in the Vedas: Vedas are store houses of all knowledge. Almost all modern scientific discoveries were already known to ancient Indians. They even knew the distance between the sun and the earth even before telescope was invented. Indians were far advanced in medicine, engineering, arts, literature, science, mathematics and every other subject. And the source of all of these was the Vedic literature. Some were directly related to the Vedas; some were taken from the Vedas and further developed by the scientists and

thinkers (Ṛṣis) of those times. The Vedas contain a great deal of knowledge on mathematics too. Exactly how old are the Vedas is not known. All that can be said is that Vedas are thousands of years old.

Knowledge of the number systems, digits, numerical codes, astronomical calculations, arithmetical calculations, progressions, geometry were covered in texts like *Taittirīya Saṁhitā*, *Ṛgveda*, *Yajurveda*, *Śatapatha Brāhmaṇa*, and so on. Some information was also blended with spiritual notes. One example would be of the Śānti Mantra[1] of *Iśāvāsya Upaniṣad* of *Yajurveda*. The mantra has a spiritual meaning, at the same time, it gives the mathematical formula and properties of Infinity as well as of Zero. In such ways, knowledge was encoded in spiritual verses in the Vedic texts.

2. Vedāṅgas and Śulba-Sūtras: Vedāṅgas are supplementary texts to understand the Vedas. There are six Vedāṅgas namely, Śikṣā (Phonetics), Niruktam (Etymology), Vyākaraṇam (Grammar), Chandas (Prosody), Kalpam (Rituals) and Jyotiṣam (Astronomy). It is to be noted that during this time, Jyotiṣam included Gaṇitam (Mathematics). In the *Chāndogya Upaniṣad*, it is given that during the late Vedic period, an integrated subject Jyotiṣam or Gaṇitam developed which was divided into three – Rāśi Vidyā (Arithmetic), Nakṣatra Vidyā (Astronomy) and Daiva Vidyā (Astrology). Kalpam was further divided into Śrauta-sutras (for sacrificial rites – yajña), Gṛhya-sūtras (domestic rules and social customs) and Dharma-sūtras (religious law). Śulba-sūtras formed a part of Śrauta-sūtras and dealt with geometrical constructions of sacrificial altars.

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1. ॐ पूर्णमदः पूर्णमिदं पूर्णात्पूर्णमुदच्यते पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते ॥ ॐ शान्तिः शान्तिः शान्तिः ॥
   Om pūrṇamadaḥ pūrṇamidaṁ pūrṇātpūrṇamudacyate pūrṇasya pūrṇamādāya pūrṇamevāvaśiṣyate. Om śāntiḥ śāntiḥ śāntiḥ.
   Meaning of the Mantra is 'That is complete. This is complete. This completeness comes from That completeness. Even if we take this completeness from That completeness, still what remains is complete.
   The Mathematical Meaning is 'When we subtract Infinity from Infinity, what remains is Infinity'. This property of Infinity also holds good for Zero. In Indian philosophy, Zero is also considered Pūrṇam (Complete).

Historians date Śulba-sūtras back to at least 1000-500 BCE. Of the many Śulba-sūtras written by different mathematicians during different periods of time, very few are available today. Among them, the well known ones are *Boudhāyana-śulba-sūtra*, *Āpastaṁba-śulba-sūtra*, *Kātyāyana-śulba-sūtra* and *Mānava-śulba-sūtra* (named after the respective authors).

3. Mathematics by Ancient Indian Mathematicians: As you have seen, the mathematics in India can be sighted in the Vedic literature which developed later into astronomy, astrology, algebra, arithmetic, geometry, and so on. As per the need of time, the ancient Indian mathematicians like Āryabhaṭa, Bhāskara, Varāhamihira, Brahmagupta, etc., compiled the existing knowledge of mathematics along with their discoveries, into both exhaustive and comprehensive treatises. Thus went on a progressive development in the field of mathematics in India.

Two of the famous schools of mathematics were Kusumapura (present day city of Patna) and Ujjain (in Madhya Pradesh). Kerala school of mathematics was also very advanced but it seems that the knowledge did not spread from this school, much, to the rest of India. However, some scholars believe that Āryabhaṭa-I (476-550 CE) was from Kerala. There were many mathematicians from Karnataka as well.

The works written by these medieval mathematicians were far advanced than the mathematics that evolved in the West. Since the last century, many Westerners have started doing research and translating many a works by the ancient Indian mathematicians.

The expositions by ancient Indian mathematicians can also be termed Vedic because it was developed from the Vedic literature.

4. Vedic Mathematics by Swami Bharati Krishna Tirtha: There lived a saint of Śaṅkarācārya order, Swami Bharati Krishna Tirtha (1884-1960 CE), who was also a contemporary, to the genius mathematician Srinivasa Ramanujan. Swamiji put forth 16 sūtras and 13 upasūtras using which calculations could

be done a lot faster. He wrote a book which was named 'Vedic Mathematics'. It is this book that is popularly known today as 'Vedic Mathematics' all over the world. However, you need to remember that Ancient Indian Mathematics (as explained in the earlier three points) does not mean only techniques to do calculations faster, but also covers the whole of mathematics as a subject.

Let us now see the history of this prevalently known 'Vedic Mathematics'.

Father of Vedic Mathematics

India has been the motherland of many great souls whose contributions have helped the whole of mankind. *Vedic Mathematics*, as the subject is called, is a gift to mankind by one of the greatest scholars India has ever produced – Swami Bharati Krishna Tirtha. He was a prodigy who left people with awe and admiration for his contribution to mathematics. He gave mathematics a whole new dimension that no one had ever imagined. He made calculations so lucid and interesting that students of any caliber can learn it with ease. He made it look like magic. The initial sense of relief that one experiences after looking through *Vedic Mathematics* is later converted into excitement that takes one onto a journey of discovery and exploration in the field of mathematics.

Born to highly learned and pious parents in 1884 CE at Tirunelveli in Madras Presidency (Tamil Nadu, India), Venkatraman (previous name of Swami Bharati Krishna Tirtha) was an exceptionally brilliant boy. He always stood first in his class for all the subjects. At the age of sixteen, he was awarded the title 'Saraswati' for his proficiency in Sanskrit. He was deeply influenced by his Sanskrit Guru Śri Vedam Venkatrai Shastri who he remembered with deepest love, reverence and gratitude. When he was twenty, he obtained an M.A. from American College of Sciences, New York (from Bombay Centre) simultaneously in seven different subjects (his subjects included Sanskrit, Philosophy, English, Mathematics, History and Science) securing the highest honours in all. He was proficient in fourteen languages.

Prof. Venkatraman Saraswati served as Principal of National College, Rajmahendri for a short period until his thirst for spiritual knowledge pulled him to Swami Sacchidananda Sivabhinava Nrisimha Bharati at Sringeri. After eight years of extensive study of the scriptures (1911-1918), Prof. Venkatraman was initiated into the Holy order of Sannyāsa as Swami Bharati Krishna Tirtha. After few years, Jagadguru Śaṅkarācārya Śri Madhusudan Tirtha's (of Govardhan Pīṭha) health took a severe turn and Swami Bharati Krishna Tirtha had to take up his position at Puri Govardhan Pīṭha.

Swamiji was a spiritual dynamo. He travelled through out India. *Vedic Mathematics* was one of his epoch-making contributions to the world. During his Vedic studies at Sringeri (1911-1918), as a result of his intense tapas (penance) in the forests of Sringeri, Swamiji unraveled the hidden meanings of certain sūtras (aphorisms or word-formulae). It is claimed that one can solve any mathematical problem using these sūtras.

About Vedic Mathematics

Vedic Mathematics is based on sixteen sūtras and thirteen upasūtras; which Swamiji says, he 're-discovered' from the Vedas, rather, some Gaṇita Sūtras from the appendix of *Atharvaveda*. One may not find these sūtras as it is in the Vedic literature, but Swamiji got the essence of the subject while he was studying Vedic literature. Being a humble saint, he did not want to claim the ownership of the new system of mathematics. Hence, he named the subject - *Vedic Mathematics*.

Initially, Swamiji had written sixteen volumes on *Vedic Mathematics* (one volume on each sūtra). But before it could be printed, the treasure was irretrievably lost. Everyone grieved over the great loss except Swamiji, for he said he could rewrite them all recollecting from his memory. In one and a half month's time he re-wrote one introductory volume. However, the toll that had taken on his failing health, on account of his rigorous work for almost four decades (and losing eyesight due to cataract) did not allow him to write more. And in February 1960, the great saint left his mortal frame.

Till date, no one has been able to find the exact source of the sūtras anywhere in the Vedic literature. For the same reason, some historians and mathematicians claim that the subject should not be called 'Vedic'. However, Vedas are very huge and some parts of the Vedic texts are not available today. The terminologies used in the sūtras by Swamiji are very similar to the ones used in the Vedic literature. It is quite possible that he had mentioned the sources of the sūtras in the volumes written by him which got lost. When we talk about the Vedas, it should be remembered that it was not written by one single person. Vedas were, in fact, revelations that the ancient Ṛṣis had. In their heights of contemplation and meditation, the knowledge of the Vedas dawned upon them. So, there is no such date when the Vedas were written. Same is said to be the case with Swami Bharati Krishna Tirtha. He was one of those kinds of Ṛṣis who gained knowledge out of intuition. How knowledge can be gained through intuition cannot be explained using logic for the simple reason that intuition is beyond logic. Since his sūtras give the results correctly, and they bear a resemblance to the Vedic terminologies, we cannot fully rule out the possibility that its source cannot be the Vedas. Further research needs to be conducted in this regard to find out the actual source of these sūtras. However, instead of getting into the dispute over the name, it would be wiser if one can make use of the knowledge, leaving the earlier part to the researchers and historians to decide.

Swami Bharati Krishna Tirtha's introductory volume on *Vedic Mathematics* covers addition, subtraction, multiplication, division, squares, square roots, cubes, cube roots, factorisation, simple and quadratic equations, H.C.F., L.C.M., decimals, fractions, and much more. The original volumes must have contained higher levels of mathematics based on the same sūtras and upasūtras. Never in the past has anyone made such a discovery by encapsulating such a vast subject like mathematics in just a few words in simply stated formulae.

The techniques used in *Vedic Mathematics* are unconventional and different from the normal mathematics that was used by earlier Indian mathematicians

like Āryabhaṭa, Bhāskara, Mahāvīra, and so on. When one hears the word 'sūtras' in a language like Sanskrit, one need not think that he should have a high-level knowledge of Sanskrit to understand the subject. As a matter of fact, the techniques prescribed are so lucid that anyone can learn it with ease. Unlike the mathematics that is taught in schools which has only 'one way' to do a particular calculation, *Vedic Mathematics* teaches different ways to solve the same problem. Perhaps, this was Swamiji's way to connect mathematics to God – Different ways to attain the same 'Truth'!

The aim of this Introductory lesson was to give you a glimpse of Indian mathematics and brief you about the subject *Vedic Mathematics* as propounded by Swami Bharati Krishna Tirtha. Now you must have understood the difference between the popularly known *Vedic Mathematics* and ancient Indian mathematics. The heritage of Indian mathematics is so vast that it cannot be encapsulated in a Course. However, while we discuss the techniques for easy calculations in the upcoming lessons, we shall take a dip into other aspects of Indian mathematics and mathematics as and when we get a chance. The interested ones can take up a few topics from the lessons for further research.

SECTION B
MULTIPLICATION WITH RECOGNIZABLE PATTERNS

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In Section A, we discussed addition and subtraction. In Section B, we shall see some high-speed techniques to multiply numbers when they appear in a certain pattern. The techniques prescribed here are very unconventional and it is difficult to find the logic behind them. But these methods give the results faster than the conventional method of multiplication if you get the numbers in those particular patterns.

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Mathematics that we learn in schools is based on logic. Anything that is not logical or which cannot be proved, is not usually accepted as a valid mathematical theory by mathematicians. For the same reason, many mathematicians have not accepted Vedic mathematics as a branch of mathematics, because according to them it is not backed by proofs. As already said in the Introductory lesson, Swami Bharati Krishna Tirtha had written 16 comprehensive volumes, which unfortunately got lost. Later, in the volume that he re-wrote, he has mentioned that it is intended only to create an interest in the readers and show what is in store for them in Vedic Mathematics in the volumes to come later. He has also mentioned that many more topics and detailed explanations would be dealt in the upcoming volumes. Sadly, he left his mortal frame before passing them on to us.

Vedic Mathematics is based on pattern-observation. Each sūtra signifies a pattern. As we go on learning the sūtras and their applications, we will see how these sūtras can be used in a large number of calculations.

Pattern-observation is very fascinating. As we learn to visualize different patterns in the world around us, we not only improve in mathematics, but also in our creative skills through creative-thinking. It is observed that young children learn many new things through this method of pattern-observation. When you see any colour or design, you relate it to some other object with a similar colour or pattern. As we get used to it, our brain automatically picks up a pattern while doing calculations too, making it a fast and easy task.
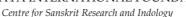
Let us look at some patterns that we come across in multiplication.

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LESSON 3
MULTIPLICATION OF COMPLEMENTARY NUMBERS

❋

Observe the following pairs of numbers to be multiplied. Do you see any particular relation in all the numbers? Think!

	35		42		77		81
x	35	x	48	x	73	x	89

Before you start reading ahead, take your time to observe the above calculations until you see patterns.

You must have observed that

1. The tens place digit is the same in both the numbers **and**
2. The units place digits add upto 10.

For such cases, **(i.e. only if both the conditions are true)** we use the below sūtras:

❋

Sūtra	**Upasūtra**
एकाधिकेन पूर्वेण	अन्त्ययोर्दशकेऽपि
Ekādhikena pūrveṇa	*Antyayordaśake'pi*
By one more than the previous one	Sum of last digits is ten

❋

$$
\begin{array}{r}
3\underline{5} \\
\times\ \underline{3}5 \\
\hline
12|25
\end{array}
\qquad
\begin{array}{r}
4\underline{2} \\
\times\ \underline{4}8 \\
\hline
20|16
\end{array}
\qquad
\begin{array}{r}
\underline{7}7 \\
\times\ \underline{7}3 \\
\hline
56|21
\end{array}
\qquad
\begin{array}{r}
\underline{8}1 \\
\times\ \underline{8}9 \\
\hline
72|09
\end{array}
$$

| 3 x (3 + 1) | 5 x 5 | | 4 x (4 + 1) | 2 x 8 | | 7 x (7 + 1) | 7 x 3 | | 8 x (8 + 1) | 1 x 9 |

To get the left side of the answer, multiply the tens place digit with one more than the same digit, i.e. if the digit in the tens place is 3, then to get the left side of the answer multiply 3 and 4.

The right side of the answer is arrived at by multiplying the units place digits. And here, we need to keep two digits. Hence, in the last example of 81 x 89 we write 09 instead of 9.

We can use the same rule for multiplying two three-digit numbers whose units place digits add upto then and the remaining digits are same, as in the case of below examples.

$$
\begin{array}{r}
10\underline{5} \\
\times\ \underline{10}5 \\
\hline
110|25
\end{array}
\qquad
\begin{array}{r}
11\underline{2} \\
\times\ \underline{11}8 \\
\hline
132|16
\end{array}
\qquad
\begin{array}{r}
50\underline{7} \\
\times\ \underline{50}3 \\
\hline
2550|21
\end{array}
$$

| 10 x (10 + 1) | 5 x 5 | | 11 x (11 + 1) | 2 x 8 | | 50 x (50 + 1) | 7 x 3 |

The same rule can be used for finding the square of numbers ending with 5. For example, 35^2, 55^2, 85^2, 105^2, and so on.

❅   ❅   ❅

Practice Problems

(1) 14 x 16   (5) 51 x 59   (9) 99 x 91   (13) 22 x 28   (17) 77 x 73

(2) 19 x 11   (6) 72 x 78   (10) 48 x 42   (14) 27 x 23   (18) 64 x 66

(3) 17 x 13   (7) 37 x 33   (11) 74 x 76   (15) 21 x 29   (19) 31 x 39

(4) 63 x 67   (8) 26 x 24   (12) 56 x 54   (16) 18 x 12   (20) 83 x 87

(21) 0.93 x 0.97 (27) 8.6 x 8.4 (33) 55^2 (39) 950^2 (45) 0.25^2

(22) 0.52 x 0.58 (28) 6.9 x 6.1 (34) 85^2 (40) 650^2 (46) 7.5^2

(23) 0.71 x 0.79 (29) 5.7 x 5.3 (35) 95^2 (41) 5.5^2 (47) 2.5^2

(24) 0.62 x 0.68 (30) 45^2 (36) 750^2 (42) 9.5^2 (48) 0.95^2

(25) 0.41 x 0.49 (31) 15^2 (37) 550^2 (43) 11.5^2 (49) 0.65^2

(26) 9.2 x 9.8 (32) 75^2 (38) 150^2 (44) 0.15^2 (50) 1.5^2

❈

Answer Set

(1) 224   (11) 5624   (21) 0.9021   (31) 225   (41) 30.25

(2) 209   (12) 3024   (22) 0.3016   (32) 5625   (42) 90.25

(3) 221   (13) 616   (23) 0.5609   (33) 3025   (43) 132.25

(4) 4221   (14) 621   (24) 0.4216   (34) 7225   (44) 0.0225

(5) 3009   (15) 609   (25) 0.2009   (35) 9025   (45) 0.0625

(6) 5616   (16) 216   (26) 90.16   (36) 562500   (46) 56.25

(7) 1221   (17) 5621   (27) 72.24   (37) 302500   (47) 6.25

(8) 624   (18) 4224   (28) 42.09   (38) 22500   (48) 0.9025

(9) 9009   (19) 1209   (29) 30.21   (39) 902500   (49) 0.4225

(10) 2016   (20) 7221   (30) 2025   (40) 422500   (50) 2.25